Bose-Einstein correlations in the target fragmentation region in 200\,A GeV $^{16}$O + nucleus collisions

WA80 Collaboration


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Abstract. Correlations between positive pions are investigated in the target fragmentation region of 200\,A GeV $^{16}$O + nucleus collisions. The pions are measured with the Plastic Ball detector in the WA80 experiment at the CERN SPS. The target mass dependence of the radii and the correlation strength extracted by interferometry is studied. A new approach to the fit of the correlation function is introduced. The correlation strength and both invariant and transverse radii increase with decreasing target mass. The transverse radius for $^{16}$O + C reactions appears to be much larger than the geometrical radius of the nuclei involved. For the Au target only a small fraction of the measured pions contributes to the apparent correlation. Hints for a much larger second component in $^{16}$O + Au reactions are observed. Rescattering phenomena may provide a clue to understand these phenomena.

1 Introduction

Pion interferometry allows a study of the space-time properties of the particle emitting source in high energy collisions [1]. An enhancement in the production of identical pions with similar momentum – known as Bose-Einstein correlations, the Hanbury-Brown–Twiss effect (HBT) [2] or the Goldhaber–Goldhaber–Lee–Pais effect (GGLP) [3] – has been observed in various experimental studies.

A simple theoretical picture of multi-particle production yields the following prediction for the two-particle correlation function [4]:

$$C_2 = \frac{\langle n \rangle^2}{\langle n(n-1) \rangle} \frac{d^6 N/dp_1^3 dp_2^3}{d^3 N/dp_1^3} \frac{d^3 N/dp_2^3}{d^3 N/dp_2^3} = 1 + \lambda \langle |\hat{p}(p_1 - p_2)\rangle^2 \rangle,$$

where $n$ is the pion multiplicity and $d^3 N/dp_1^3$ and $d^3 N/dp_2^3$ are the one-pion and two-pion inclusive yields, $\hat{p}$ is the Fourier transform of the distribution of emitters; the most commonly chosen analytic expressions for $\hat{p}$ contain a suitable correlation length $R$. While this parameter seems to measure dynamical properties of the produced strings in $e^+e^-$ collisions [5], it can provide geometrical information in medium energy heavy ion collisions ($E_{lab} \leq 2A$ GeV) [6].

The parameter $\lambda$, a correlation strength, was introduced for technical reasons [7]; it is expected to be $\approx 1$ for a completely chaotic source. Theoretically a value of $\lambda < 1$ can be ascribed to a certain amount of coherent production of pions [8], but in the experiment many different effects may reduce the measured value of $\lambda$.

The existence of a deconfined phase with high energy density but small pressure gradient in nuclear collisions would lead to a considerably larger lifetime of the system. It has been suggested in [9] that such an effect could...
be observed by pion interferometry and would thus be a possible signal of the quark-gluon plasma.

These theoretical investigations as well as the experimental observation of a large source at midrapidity ($R_\pi = 7-8$ fm) by the NA35 collaboration in $^{16}\text{O}+\text{Au}$ collisions at 200 GeV [10] have renewed the interest in interferometric measurements. They can provide an important clue to understand the dynamics of ultrarelativistic nucleus-nucleus collisions. In this paper we will present a study of $\pi^+\pi^-$-correlations in reactions of $200\text{A GeV} \ 16\text{O}+\text{Au}$ nucleus in the target rapidity region ($y_{\text{lab}} \leq 1$).

The importance of rescattering of secondary particles in the target spectator matter in these reactions has been emphasized in an earlier publication [11]. It is therefore of interest to study pion correlations in the target fragmentation region to clarify both the general role of the target spectators and their influence on pion interferometry. In addition, interferometry in the target rapidity region complements the data for $^{18}\text{O}+\text{Au}$ collisions near mid-rapidity [10].

2 Data analysis

Data were taken in the WA80 experiment [12] at the CERN SPS during the $^{16}\text{O}$ run in fall 1986. The pions were identified with the plastic ball detector [13], a sphere of 655 individual modules, in $30^\circ \leq \theta_{\text{lab}} \leq 160^\circ$. Each module consists of a slow CaF$_2$ $\Delta E$-counter and a fast plastic scintillator to measure the remaining energy. Both scintillators are optically coupled to one phototube and read out in a Phoswich technique. These telescopes are capable of identifying charged fragments up to helium isotopes. Pions can be identified with the $\Delta E$-$E$-technique in the kinetic energy range 20 MeV $\leq E_{\text{kin}} \leq 120$ MeV.

In the high multiplicity environment of the investigated reactions the identification of pions by energy loss has to suffer a background from high energy protons, which are not stopped in the detector, and from neutral particles. Therefore the delayed signal of the sequential decay

$$\pi^- \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \nu_e \nu_\mu \bar{\nu}_\mu$$

was used in addition to make the identification unique and to prevent a masking of the pions by baryons. $\pi^-$ does not show decay signal, because they are promptly absorbed by nuclei in the detector. The efficiency of this identification was measured in a calibration and is between 50% and 80% depending on the pion energy [13].

Particular attention was paid to the problems which can arise from positrons scattering out of a scintillator. As was pointed out in [14], the positron will often penetrate into an adjacent module coinciding with other particles and thereby simulating the $\pi^-$-signal. This yields fake pion pairs with dominantly small angular separation, which would lead to an enhancement in the correlation function for small relative momenta. The fake pion pairs have coincident decay-time signals, because these originate from the same positron. They can therefore be excluded by requiring that no two $\pi^+$ show time signals which satisfy $\Delta t = |t_1 - t_2| \leq 60$ ns. In the analysis presented here all such fake pions have been removed with the help of this cut on the time difference $\Delta t$.

This time cut however introduce another small bias in the correlation function. A fraction of the correlated pairs with coincident decay-time signals may in fact be true $\pi^+$-pairs, where the time signal of one particle is suppressed by the scattered positron from the decay of the other, if this decay occurs earlier. This loss of true pairs would cause a possible correlation to appear weaker than it really is. This efficiency is expected to be mainly dependent on the penetration probability of a positron into a neighboring module, i.e. on detector properties and single particle and two particle variables of the pions (e.g. $\pi^+$ kinetic energy or $\pi^+\pi^+$ opening angle), not on multiplicity.

It can be seen from the data that these problems are present only for pairs of detector modules close by. For modules having an angular separation larger than $\psi_{\text{min}} = 25^\circ$ this "cross-talk" is totally negligible. All systems have been analyzed with such an additional angular cut, so that no efficiency problems are left. However, this cut imposes a severe restriction in pair acceptance for small relative momenta. This should be kept in mind, if the measured values are compared to those from other experiments.

The point of impact of each particle was chosen randomly on the surface of the corresponding module. In a Monte-Carlo simulation taking into account the measured energy resolution and the above mentioned angular smearing the resolution in relative momentum is estimated to be $\delta q \approx 10$ MeV/c, where the contribution of the intrinsic energy resolution is negligible. The effective energy resolution is worsened by multiple hits occurring in high multiplicity events. A double hit of a pion with another particle may have two consequences. Either the second particle releases a large amount of energy in the scintillators and the pion is rejected, because the signals don't match the expected energy loss, or only a small amount of energy is added to the measured pion energy - this worsens the energy resolution.

The effects of such double hits were evaluated in a Monte-Carlo simulation. One well identified $\pi^+$ from a low multiplicity event was added to one event of the very highest multiplicity (raw multiplicity in the plastic ball $M_{\text{raw}} > 300$). The ADC-values for all modules were added individually and the same particle identification algorithm as for real events was used. The change in efficiency for the individual pions could thus be obtained.

A final simulation step was then used to estimate the influence of resolution and multiple hits on the measured correlation functions. Pairs of pions were generated according to the observed single particle phase space distributions and weighted with a correlation function of the form:

$$C_2(Q) = 1 + \lambda \exp (-\frac{1}{2} Q^2 R^2).$$

(2)

A detector filter was applied and the above mentioned cut in opening angle was imposed. Correlation functions
were then calculated from the simulated data and fitted with (2). Figure 1 shows the extracted parameters $R_{\text{fit}}$ and $\lambda_{\text{fit}}$ as a function of the input radius $R_{\text{in}}$ for $\lambda_{\text{in}} = 1$. The open squares show the pure effects of detector resolution including the angular cut, for the circles multiple hits for high multiplicity are added. It can be seen that even with double hits in the worst case assumption rather large radii can still be measured, the extracted values being only slightly too small. For the large radii the fitted $\lambda$-parameter however drops considerably. The uncertainty on the parameters increases as well, a fact that is connected to the angular cutoff. Similar effects are observed for transverse and longitudinal radii.

The quality of the $\pi^+$-identification was then estimated from the constant background in the exponential decay time spectra. The contamination of misidentified particles $x_{\text{mis}}$ is given in Table 1. It is less than 4% for all samples. Additional details on the detector design and performance are given in [13].

The acceptance of the plastic ball is given by the energy limits and the angular coverage; it corresponds to rapidities $y_{\text{lab}} \leq 1$ and transverse momenta $p_T \leq 220 \text{ MeV/c}$. Figure 2 shows contour lines of the number of pions as a function of $p_T$ and $y_{\text{lab}}$ for central collisions of $^{16}\text{O} + \text{Au}$. Within the acceptance the shape of the distribution is nearly identical for all investigated systems.

Reactions with 200.4 GeV $^{16}\text{O}$ projectiles and C, Cu, Ag and Au targets were investigated. The selection of events with at least two well identified positive pions in the target region introduces a bias towards central events. In the WA80 experiment the centrality may however be determined independently from the forward energy $E_F$ measured in the Zero-Degree-Calorimeter [15].

It has been shown [16] that global variables like $E_F$ are reasonably well described by models of independent nucleon-nucleon collisions. We have therefore used the FRITIOF event-generator [17] to extract from $E_F$ the number of participating baryons. The average number of participants $\langle N_{\text{part}} \rangle$ for each sample is given in Table 1. The table also contains the total number of $\pi^+$-pairs used in the analysis, the average total raw multiplicity in the plastic ball $\langle M_{\text{PB}} \rangle$, the average multiplicity of identified positive pions $\langle M_{\pi^+} \rangle$ and the fraction of misidentified particles $x_{\text{mis}}$.

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The experimental two-pion correlation function was evaluated by taking the ratio of the distributions of correlated and uncorrelated pairs. Uncorrelated pairs were obtained by the so-called “event-mixing” procedure: A pion from one event was combined with a pion from another event with the same apparent $\pi^+$-multiplicity.

To check the origin of possible pair correlations we have calculated correlation functions, where one $\pi^+$ was

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Table 1. Summary of $\pi^+$-pair samples. Given are the mean number of participants $\langle N_{\text{part}} \rangle$ extracted with the help of the FRITIOF Monte-Carlo, the number of pion pairs used, the average total raw multiplicity in the plastic ball $\langle M_{\text{PB}} \rangle$, the average multiplicity of identified positive pions $\langle M_{\pi^+} \rangle$ and the fraction of misidentified particles $x_{\text{mis}}$.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$^{16}\text{O} + \text{Au}$</th>
<th>$^{16}\text{O} + \text{Ag}$</th>
<th>$^{16}\text{O} + \text{Cu}$</th>
<th>$^{16}\text{O} + \text{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle N_{\text{part}} \rangle$</td>
<td>52.9</td>
<td>47.2</td>
<td>39.5</td>
<td>19.2</td>
</tr>
<tr>
<td>$\pi^+$ pairs/10$^3$</td>
<td>2193</td>
<td>429</td>
<td>365</td>
<td>30</td>
</tr>
<tr>
<td>$\langle M_{\text{PB}} \rangle$</td>
<td>241</td>
<td>173</td>
<td>132</td>
<td>56</td>
</tr>
<tr>
<td>$\langle M_{\pi^+} \rangle$</td>
<td>4.7</td>
<td>4.0</td>
<td>3.5</td>
<td>2.5</td>
</tr>
<tr>
<td>$x_{\text{mis}}$</td>
<td>3.4%</td>
<td>2.2%</td>
<td>2.1%</td>
<td>3.5%</td>
</tr>
</tbody>
</table>

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Fig. 1. Parameters $R_{\text{fit}}$ and $\lambda_{\text{fit}}$ extracted from simulated correlation functions as a function of the input radius $R_{\text{in}}$. The simulation includes detector resolution (open squares) and resolution with additional influence of multiple hits for very high multiplicity events (open circles).

Fig. 2. Contour lines of accepted pion pairs (linear scale) as a function of $p_T$ and $y_{\text{lab}}$ for central collisions of $^{16}\text{O} + \text{Au}$.
For simplicity, Gaussian functions have been used both for one- and two-dimensional analysis:

\[ C_2(Q) = 1 + \lambda_{\text{inv}} \exp \left( -\frac{1}{2} \left( R_{\text{inv}}^2 Q^2 \right) \right) \]  
\[ C_2(Q_r, Q_L) = 1 + \lambda_{\text{lab}} \exp \left( -\frac{1}{2} \left( R_r^2 Q_r^2 - \frac{1}{2} R_L^2 Q_L^2 \right) \right). \]

For the analysis in \( Q \) an exponential function:

\[ C_2(Q) = 1 + \lambda_{\text{inv}} \exp \left( -\frac{1}{2} R_{\text{inv}}^2 Q^2 \right) \]  

has also been used for comparison. Data were fitted in the region of \( Q \leq 400 \text{ MeV} \) by maximizing the logarithm of a likelihood function \( \mathcal{L} \). The negative logarithm of the likelihood function has been normalized in such a way, that it should tend towards a \( \chi^2 \)-value in the limit of large statistics, i.e. good fits should also give a value of \( -\log \mathcal{L} \) close to the number of degrees of freedom \( v \). For the fits in \( (Q_r, Q_L) \) the energy difference was always \( q_0 = |E_1 - E_2| \leq 100 \text{ MeV} \).

One of our data samples could not be fitted with the simple one-dimensional correlation functions. We have therefore investigated a different approach. The correlation function can be decomposed in the following way:

\[ C_2(Q) = 1 + \sum_{i=1}^{N} x_i \tilde{p}_i(Q) \]

The functions \( \tilde{p}_i(Q) \) contain radius parameters \( (R_i; r_i) \), which define the range of source radii accessible with this method. Within this range very different shapes can however be described by varying the \( x_i \). From the fits of these correlation functions rms-radii and a correlation strength parameter \( \xi \) can be extracted. Details about the two versions of these functions used here (multi-Gaussian and multi-sine) can be found in the Appendix.

### 3 Results

Figure 4a–d show \( \pi^+ \pi^+ \)-correlations as a function of \( Q \) for central reactions of \( 200A \text{ GeV} \) \( ^{16}O \) with different targets in the rapidity region \(-1 \leq y_{\text{lab}} \leq 1\). A clear enhancement for small values of \( Q \) is visible in all cases. The fits with a Gaussian parametrization are shown as a solid, the exponential fits as a dashed line. The two other lines represent the superposition of basis functions explained above, the dotted line showing the Gaussian ansatz, the dash-dotted the one employing the sine expressions. For the system \( ^{16}O + C \) no stable fits could be obtained for the simple Gaussian or exponential functions, so only the other two fits are given. The resulting fit parameters are summarized in Table 2. For the \( \text{Au} \) and \( \text{Ag} \) targets the exponential form provides the best fit to the data. The multi-Gaussian is only slightly worse – the main differences in these two forms can be seen in the region of very small \( Q \) outside the range of the experimental data. For the \( \text{Cu} \) target the single Gaussian and the multi-sine functions are both slightly preferable.
Fig. 4a–d. $\pi^+\pi^+$ correlation (Gamow-corrected) as a function of $Q$ for central reactions of 200A GeV $^{16}$O with various targets in the rapidity region $-1 \leq y_{lab} \leq 1$. The lines show different fits to the correlation function (see text). a Au, b Ag, c Cu and d C target.
Table 2. Pion correlation fit parameters in central reactions for $-1 < \gamma_{\pi\gamma} < 1$ extracted by Gaussian and exponential fits and results of the fits of the two sets of basis functions as explained in the text

<table>
<thead>
<tr>
<th>Target</th>
<th>Au</th>
<th>Ag</th>
<th>Cu</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-\log L\Phi/v$</td>
<td>58.7/35</td>
<td>69.9/35</td>
<td>53.5/35</td>
<td>-</td>
</tr>
<tr>
<td>$R_{\mathrm{inv}}^{\mathrm{GM}}$ (fm)</td>
<td>2.38 ± 0.08</td>
<td>3.45 ± 0.15</td>
<td>3.33 ± 0.15</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{\mathrm{inv}}$</td>
<td>0.085 ± 0.007</td>
<td>0.17 ± 0.04</td>
<td>0.17 ± 0.03</td>
<td>-</td>
</tr>
<tr>
<td>$R_{\mathrm{inv}}^{\mathrm{GM}}$ (fm)</td>
<td>2.91 ± 0.12</td>
<td>4.23 ± 0.18</td>
<td>4.08 ± 0.18</td>
<td>-</td>
</tr>
<tr>
<td>Exponential</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-\log L\Phi/v$</td>
<td>46.5/35</td>
<td>61.5/35</td>
<td>56.3/35</td>
<td>-</td>
</tr>
<tr>
<td>$R_{\mathrm{inv}}^{\mathrm{EP}}$ (fm)</td>
<td>3.07 ± 0.12</td>
<td>5.46 ± 0.24</td>
<td>5.05 ± 0.25</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{\mathrm{inv}}$</td>
<td>0.20 ± 0.03</td>
<td>0.59 ± 0.21</td>
<td>0.49 ± 0.14</td>
<td>-</td>
</tr>
<tr>
<td>Multi-Gaussian</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-\log L\Phi/v$</td>
<td>45.8/32</td>
<td>62.2/32</td>
<td>53.8/32</td>
<td>66.5/32</td>
</tr>
<tr>
<td>$\xi^2$</td>
<td>0.14</td>
<td>0.25</td>
<td>0.17</td>
<td>1.06</td>
</tr>
<tr>
<td>$R_{\mathrm{inv}}^{\mathrm{MG}}$ (fm)</td>
<td>3.80</td>
<td>4.67</td>
<td>4.13</td>
<td>6.09</td>
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<tr>
<td>Multi-sine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-\log L\Phi/v$</td>
<td>51.2/32</td>
<td>65.9/32</td>
<td>50.5/32</td>
<td>72.1/32</td>
</tr>
<tr>
<td>$\xi^2$</td>
<td>0.10</td>
<td>0.16</td>
<td>0.15</td>
<td>0.68</td>
</tr>
<tr>
<td>$R_{\mathrm{inv}}^{\mathrm{MS}}$ (fm)</td>
<td>1.88</td>
<td>2.23</td>
<td>2.03</td>
<td>2.76</td>
</tr>
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</table>

Fig. 5. Fit parameters ($R_{\mathrm{inv}}$, $\lambda_{\mathrm{inv}}$) of the $2\pi^+$ correlation as a function of the rms target radius. The open circles show results from Gaussian fits and the open squares those from exponential fits.

The one-dimensional correlation functions have also been analyzed without the angular cutoff mentioned above. Figure 7 shows such a correlation function for $^{16}\text{O}+\text{Au}$ collisions. It can be seen that the data now reach out to smaller values of $Q$. One should however keep in mind that there is a slight decrease in pion pair efficiency for small $Q$ in this case due to the time cut.

The magnitude of this efficiency variation may be estimated by comparing correlation functions obtained for the two cases. Figure 8 shows the ratio of the correlation function with the angular cut to the function without the cut in their region of overlap. As expected there is a small rise in this ratio for $Q \leq 60$ MeV. One can conclude that the correlation functions obtained without the angular cut should only be slightly too low for $Q \leq 60$ MeV. It should be noted however that the $Q$-space is populated differently in the two samples because of the cut in opening angle.

Table 3 contains the fit parameters obtained by Gaussian and exponential fits to these data. The radii are comparable to the previous analysis (Table 2), in most cases the $\lambda$-parameter is slightly smaller, as expected. Figure 9 displays the target dependence of the parameters.

These results confirm the counterintuitive observation that the radii $R_{\mathrm{inv}}$ decrease with increase target mass. Also the intercept $\lambda_{\mathrm{inv}}$ goes down for the heavier systems.

In addition transverse and longitudinal radii have been extracted. Here the angular cut was again applied. Figure 10a–d shows projections of the correlation functions for the various targets as a function of $Q_L$ and $Q_T$. The systematic comparison of the correlation functions shows that the exponential form has always the highest intercept at $Q = 0$, the Gaussian the lowest. The two other functions have a behaviour intermediate between the two extreme cases. These methods can therefore be used to relate the $^{16}\text{O}+\text{C}$ data set to the other projectile-target combinations, although the errors on the extracted radii are very difficult to obtain.

We have checked the systematic uncertainty in these methods by varying the exact values of the $R_i$ and $r_i$ slightly; the differences are found to be negligible.

In Fig. 5 the radii $R_{\mathrm{inv}}$ and the intercept $\lambda_{\mathrm{inv}}$ as a function of the rms target radius $R_{\mathrm{target}}^{\text{rms}} = \sqrt{\frac{3}{5}} (1.128A_{\text{target}}^{1/3} + 2.24A_{\text{target}}^{1/3})$ fm $[18]$ are displayed. The circles show the parameters for the Gaussian, the squares those for the exponential fits. While the Ag and Cu targets give similar results, both parameters are much smaller for the Au target. In addition the extracted values as a function of the target radius for the two superposition methods (see appendix) are shown in Fig. 6a and b, where the C target was added. No error bars can be given here, but the different symbols represent results for sets of $R_i$ and $r_i$ differing by 0.1 fm. These variations do not change the overall target dependence. The fact that the multi-Gaussian fits yield radii which are about a factor of two larger than those for the multi-sine method is related to the larger range of sensitivity of the first method. This is also reflected by the larger values of $\xi$ – a larger fraction of the source can be accounted for.

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Fig. 6a, b. Extracted parameters ($R_{\text{rms}}^\text{ms}$, $\xi$) of the $2\pi^+$ correlation as a function of the rms target radius. The different symbols show results for small variations of the basis functions (see text). a Multi Gaussian and b multi-sine fits.

Fig. 7. $\pi^+\pi^+$ correlation (Gamow-corrected) as a function of $Q$ for central reactions of 200A GeV $^{16}$O + Au in the rapidity region $-1 \leq y_{lab} \leq 1$. No cut in opening angle was imposed. The solid line represents a Gaussian, the dashed line an exponential fit. The inset shows the region $Q \leq 70$ MeV in a different scale.

Fig. 8. Ratio of the correlation function obtained with the angular cutoff to the one without the cut for $^{16}$O + Au collisions.
Table 3. Pion correlation fit parameters in central reactions for $-1 \leq \gamma_{lab} \leq 1$ extracted by Gaussian and exponential fits. No cut on the opening angle was imposed.

<table>
<thead>
<tr>
<th>Target</th>
<th>Au</th>
<th>Ag</th>
<th>Cu</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>$-\log L\mathcal{F}/\gamma$</td>
<td>$R^{\text{av}}_T$ (fm)</td>
<td>$R^{\text{inv}}_T$ (fm)</td>
<td>$\lambda^{\text{inv}}_T$</td>
</tr>
<tr>
<td></td>
<td>124.9/37</td>
<td>70.6/37</td>
<td>71.3/37</td>
<td>52.2/37</td>
</tr>
<tr>
<td></td>
<td>$2.5 \pm 0.1$</td>
<td>$3.1 \pm 0.2$</td>
<td>$3.5 \pm 0.1$</td>
<td>$4.1 \pm 0.3$</td>
</tr>
<tr>
<td></td>
<td>$0.094 \pm 0.003$</td>
<td>$0.14 \pm 0.01$</td>
<td>$0.16 \pm 0.01$</td>
<td>$0.40 \pm 0.04$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$-\log L\mathcal{F}/\gamma$</td>
<td>$R^{\text{av}}_T$ (fm)</td>
<td>$R^{\text{inv}}_T$ (fm)</td>
<td>$\lambda^{\text{inv}}_T$</td>
</tr>
<tr>
<td></td>
<td>109.9/37</td>
<td>79.2/37</td>
<td>84.6/37</td>
<td>44.3/37</td>
</tr>
<tr>
<td></td>
<td>$2.5 \pm 0.1$</td>
<td>$3.3 \pm 0.4$</td>
<td>$4.2 \pm 0.3$</td>
<td>$5.7 \pm 0.6$</td>
</tr>
<tr>
<td></td>
<td>$0.162 \pm 0.006$</td>
<td>$0.22 \pm 0.03$</td>
<td>$0.29 \pm 0.03$</td>
<td>$0.92 \pm 0.10$</td>
</tr>
</tbody>
</table>

Table 4. Pion correlation fit parameters in central reactions for $-1 \leq \gamma_{lab} \leq 1$ extracted by two-dimensional Gaussian fits.

<table>
<thead>
<tr>
<th>Target</th>
<th>Au</th>
<th>Ag</th>
<th>Cu</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-\log L\mathcal{F}/\gamma$</td>
<td>$R^{\text{av}}_T$ (fm)</td>
<td>$R^{\text{av}}_L$ (fm)</td>
<td>$\lambda^{\text{inv}}_T$</td>
</tr>
<tr>
<td></td>
<td>2546/1008</td>
<td>1309/992</td>
<td>1122/990</td>
<td>1045/882</td>
</tr>
<tr>
<td></td>
<td>$2.40 \pm 0.15$</td>
<td>$3.01 \pm 0.34$</td>
<td>$3.22 \pm 0.26$</td>
<td>$3.69 \pm 0.57$</td>
</tr>
<tr>
<td></td>
<td>$1.73 \pm 0.12$</td>
<td>$3.89 \pm 0.45$</td>
<td>$3.57 \pm 0.50$</td>
<td>$2.82 \pm 0.65$</td>
</tr>
<tr>
<td></td>
<td>$0.081 \pm 0.004$</td>
<td>$0.18 \pm 0.02$</td>
<td>$0.18 \pm 0.02$</td>
<td>$0.37 \pm 0.09$</td>
</tr>
</tbody>
</table>

Figure 9. Fit parameters ($R^{\text{av}}_T$, $R^{\text{inv}}_T$, $\lambda^{\text{inv}}_T$) of the $2\pi^+$ correlation as a function of the rms target radius. The open circles show results from Gaussian fits and the open squares those from exponential fits. No cut in opening angle was imposed.

$Q_T$ together with Gaussian fits. Because of the limits of statistics for such a two-dimensional analysis no further investigation of different analytical forms has been attempted. Table 4 contains the extracted parameters for the different systems.

Figure 11 displays the results as a function of the target radius. One clearly sees that the transverse radii and the correlation strength decrease with increasing target size. The longitudinal radii show a more complicated behaviour. There is an increase in $R_T$ for the three lighter targets, only the Au target shows a much smaller value.

The very high statistics of the system $^{16}$O + Au allows a more detailed analysis. Transverse radii have been extracted for various cuts in $Q_L$. While most of the parameters are similar to those obtained with the two-dimensional fit, for $Q_L \leq 10$ MeV/c the following values have been extracted with a Gaussian fit: $R_T = 4.95 \pm 0.16$ fm and $\lambda = 0.42 \pm 0.10$. The fit has a likelihood value of $-\log L = 62.7$ for $v = 27$ degrees of freedom. The corresponding correlation function is shown in Fig. 12 as a solid line.

This observation indicates that the correlation function in this case can not be described by single transverse and longitudinal radii in a gaussian parametrization. Therefore fits of two-component correlation functions in two dimensions ($Q_T$ and $Q_L$) have been attempted. If one assumes that two different sources contribute only in different kinds of events, a sum of two correlation functions would be expected:

$$C_2(Q_T, Q_L) = 1 + \lambda_1 \cdot \exp \left( -\frac{Q_T^2 R_{T1}^2 - Q_L^2 R_{L1}^2}{2} \right) + \lambda_2 \cdot \exp \left( -\frac{Q_T^2 R_{T2}^2 - Q_L^2 R_{L2}^2}{2} \right).$$

(6)
Fig. 10a–d. Projected π⁺π⁺ correlation function (Gamow-corrected) for central reactions of 200 A GeV ¹⁶O with different targets in the rapidity region $-1 \leq y_{ab} \leq 1$. The lines show Gaussian fits to the correlation functions. The upper figure shows the correlation for $Q_T \leq 50$ MeV/c as a function of $Q_L$, the lower one for $Q_L \leq 50$ MeV/c as a function of $Q_T$. a Au, b Ag, c Cu and d C target.
Fig. 11. Fit parameters ($R_T$, $R_L$, $\lambda_{lab}$) of the $\pi^+\pi^+$ correlation as a function of the rms target radius.

Table 5. Pion correlation fit parameters in central $^{16}\text{O} + \text{Au}$ reactions for $-1 \leq y_{lab} \leq 1$ extracted by two-dimensional Gaussian fits with two components.

<table>
<thead>
<tr>
<th>Parametrization</th>
<th>Equation (6)</th>
<th>Equation (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\log \mathcal{L}/\mathcal{L}^*$</td>
<td>2489/1008</td>
<td>2486/1008</td>
</tr>
<tr>
<td>$R_{T1}$ (fm)</td>
<td>2.28 ± 0.10</td>
<td>2.10 ± 0.13</td>
</tr>
<tr>
<td>$R_{L1}$ (fm)</td>
<td>1.56 ± 0.10</td>
<td>1.34 ± 0.12</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.072 ± 0.004</td>
<td>0.246 ± 0.010</td>
</tr>
<tr>
<td>$R_{T2}$ (fm)</td>
<td>6.57 ± 0.28</td>
<td>7.10 ± 0.38</td>
</tr>
<tr>
<td>$R_{L2}$ (fm)</td>
<td>19.9 ± 4.1</td>
<td>10.3 ± 1.9</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.67 ± 0.25</td>
<td>0.36 ± 0.08</td>
</tr>
</tbody>
</table>

If both sources are present in all events, the function would be:

$$C_2(Q_T, Q_L) = 1 + \lambda_1 \cdot \exp \left( \frac{-Q_T^2 R_{T1}^2 - Q_L^2 R_{L1}^2}{4} \right)$$

$$+ \lambda_2 \cdot \exp \left( \frac{-Q_T^2 R_{T2}^2 - Q_L^2 R_{L2}^2}{4} \right)^2.$$  \hspace{1cm} (7)

Both parametrizations yield better fits than the single Gaussian – (7) gives only a slightly better agreement than (6). The fit results are summarized in Table 5. The projections of these correlation functions for $Q_L \leq 10$ MeV/c are displayed in Fig. 12. In both cases the extracted parameters show a small component with $R_{T1} \approx 2$ fm and $R_{L1} \approx 1.5$ fm and a much larger component with $R_{T2} \geq 6$ fm and $R_{L2} \geq 10$ fm. On the one hand these large radii will necessarily be influenced by the detector resolution – they may rather provide lower limits on the true values. On the other hand the influence of the Coulomb correction and possible systematic errors of it should not be very important, as the cut in opening angle described above forces the pairs to have finite relative momentum in their center of mass $Q \geq 30$ MeV.

4 Discussion

How can the variations in the parameters be understood in our current picture of ultrarelativistic nuclear collisions? As has been pointed out, a reduction of the value of $\lambda$ may be caused by a variety of mechanisms. Most of these, as e.g. final state interaction or resonance production, should not show such a drastic variation for different targets.
It is however easy to realize that pions observed in the target rapidity region might undergo very different evolutions. They may be

- directly produced by the participant nucleons, traversing the target nucleus without further interaction, or
- rescattered in the target spectator matter after their production.

It has been noted that data from the target fragmentation region indicate rescattering effects [11]. Such rescattering need not affect the pion multiplicities, but it would alter the volume measured by interferometry to the “freeze-out” volume of the excited spectator matter. The rescattered pions might come from very large volumes or late times, and their correlation could thus be hidden at very small values of $Q$ hardly accessible to experiment. Radii larger than 20 fm could not be measured with the present analysis.

There is an indication of a steep rise in the correlation function for $^{16}$O + Au analyzed without angular cut (Fig. 7), which contributes partially to the bad likelihood value of the corresponding fits. Because of the uncertainties related to the unknown efficiency in this case and to the sensitivity of the first data points to effects of the Coulomb interaction no further information can be obtained at this stage of the analysis.

The effects of sources of various sizes on the apparent strength of the interference has been discussed earlier [19]. Here we will concentrate on the case of two sources with different radii, one of them being much larger than the other. The effects can be demonstrated by simulating examples of correlation functions. Pion pair distributions have been weighted with the following function:

$$C_2(Q) = 1 + \lambda_1 \cdot \exp \left( -\frac{Q^2 R_1^2}{2} \right) + (1 - \lambda_1) \cdot \exp \left( -\frac{Q^2 R_2^2}{2} \right).$$

(8)

Correlation functions have been constructed and fitted with a single Gaussian (3) or exponential (5). The extracted parameters for five sets of input values are given in Table 6.

The radii and the $\lambda$-parameter extracted from this simulation decrease, when both the size of the large component ($R_2$) and its relative contribution ($1 - \lambda_1$) increase. The influence of a second component on the measured parameters is negligible, if the second radius is much larger than the first one. This behaviour may account for the target dependence observed in the data.

More generally this illustrates that, if the measured source does not obey the distribution one chooses to fit the correlation function, one might miss the part of the correlation function where the statistical errors allow no decision. Thus the apparent correlation strength is reduced, just like it would be in the case of bad detector resolution.

The existence of a second effective source of pions consisting of the target spectator matter could explain the qualitative target dependence of the invariant and transverse radii and the correlation strength $\lambda$. As $\lambda$ is not a very well understood parameter in pion interferometry, there may still be other explanations for its behaviour as indicated above. For the radii however no other natural explanation seems to be at hand.

The invariant radii are not easily interpreted in geometrical terms, because they contain both spacelike and timelike information. The interpretation of transverse radii should not suffer from this, and they are also free from influence of boosts in the beam direction.

The transverse radius for $^{16}$O + Au reactions ($R_T = 2.4$ fm) compares very well to the projectile radius $R(16O) = (2.3 \pm 0.3)$ fm, which is the Gaussian equivalent of the rms charge radius $R_{rms}(^{16}O) = (2.8 \pm 0.3)$ fm [20]. In this case the observed correlation seems to originate dominantly from pions produced directly in the geometrical overlap area. The value of $R_T$ for the C target ($R_T = 3.7$ fm) is considerably larger than both the projectile and the target radii. So in the spectator-free system there is an indication of transverse expansion prior to freeze-out.

The longitudinal radii for the three lighter targets show an increase with target mass consistent with a geometrical interpretation. The value of $R_L$ for the Au target is however much smaller. These radii might be strongly influenced by Lorentz transformations, but e.g. a transformation of $R_L$ measured in the lab to the average center of mass system would rather reduce the radii further. While the extracted parameters for C, Cu and Ag can be understood as showing a source locally at rest in the target system extending over almost the whole target nucleus, the data for $^{16}$O + Au do not fit into the same picture. A detailed understanding of the longitudinal expansion is obviously necessary. One should note that there is a hidden influence of the source lifetime in the extraction of $R_T$ and $R_L$. By integrating the data sample over $q_0 \lesssim 100$ MeV one has implicitly chosen a range of lifetimes $\tau$. Therefore this analysis of transverse and

### Table 6. Pion correlation fit parameters for simulated two-component correlation functions (8) extracted by single Gaussian and exponential fits

<table>
<thead>
<tr>
<th>$R_1$ (fm)</th>
<th>$R_2$ (fm)</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$R_{rms}$ (fm)</th>
<th>$\lambda_{rms}$</th>
<th>$R_{exp}$ (fm)</th>
<th>$\lambda_{exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>0.09</td>
<td>0.09</td>
<td>2.34 ± 0.06</td>
<td>0.090 ± 0.003</td>
<td>2.37 ± 0.13</td>
<td>0.155 ± 0.006</td>
</tr>
<tr>
<td>2.5</td>
<td>18</td>
<td>0.105</td>
<td>0.105</td>
<td>2.59 ± 0.14</td>
<td>0.136 ± 0.010</td>
<td>2.80 ± 0.32</td>
<td>0.242 ± 0.022</td>
</tr>
<tr>
<td>2.5</td>
<td>16</td>
<td>0.135</td>
<td>0.135</td>
<td>3.05 ± 0.25</td>
<td>0.175 ± 0.017</td>
<td>4.00 ± 0.42</td>
<td>0.38 ± 0.04</td>
</tr>
<tr>
<td>2.5</td>
<td>14</td>
<td>0.15</td>
<td>0.15</td>
<td>3.24 ± 0.13</td>
<td>0.257 ± 0.013</td>
<td>4.76 ± 0.20</td>
<td>0.62 ± 0.03</td>
</tr>
<tr>
<td>2.5</td>
<td>10</td>
<td>0.20</td>
<td>0.20</td>
<td>3.69 ± 0.24</td>
<td>0.49 ± 0.04</td>
<td>5.77 ± 0.47</td>
<td>1.34 ± 0.15</td>
</tr>
</tbody>
</table>
longitudinal components is not sensitive to parts of the source with \( \tau \gg \hbar c/q_0 = 2 \text{ fm}/c \).

In addition, a two-dimensional fit like the one employed here will provide only average radii, if the two directions do not factorize. A fit to the transverse direction with a very stringent cut on the longitudinal component \( Q_L \leq 10 \text{ MeV}/c \) yields indeed a significantly larger radius, \( R_T = 5 \text{ fm} \). One should note again that the larger value is only observed for this specific cut in \( Q_L \). This shows that the longitudinal radius of this component of the source is obviously very large.

This is corroborated by the two-component fits ((6) and (7)). While the smaller component is again compatible with the transverse size of the \(^{16}\text{O}\) nucleus, there is a larger component with \( R_{T2} \geq 6 \text{ fm} \) and \( R_{L2} \geq 10 \text{ fm} \).

The rather low values of \( \lambda \) indicate that only a small fraction of the measured pions contributes to the observed correlation. In the interpretation sketched above this is attributed to the fact that for heavier targets a large fraction of the produced particles undergoes rescattering processes in the spectator matter. The effective source of these rescattered pions is then hardly accessible in the present analysis.

5 Summary and conclusions

We have observed correlations of pairs of positive pions in the target fragmentation region for central regions of \( 200A \text{ GeV} \) \(^{16}\text{O}\) on \( \text{C, Cu, Ag and Au} \) targets. The interpretation of the enhancement in terms of Bose-Einstein correlations yields the following results:

1. There is no simple optimal analytic expression for the one-dimensional correlation functions of all investigated systems. For the heavier targets an exponential seems to provide the best fit to the data.
2. The extracted radii \( R_{\text{inv}} \) and \( R_T \) and the correlation strength \( (\lambda_{\text{inv}}, \lambda_{\text{lab}}) \) decrease with increasing target mass. Qualitatively this effect does not depend on the explicit choice of the parametrization. One possible explanation of this observation may be the existence of a second large component of the source. Such a second source might be caused by rescattering of produced particles in the target spectator matter.
3. In \(^{16}\text{O} + \text{Au}\) central collisions the average transverse radius can be interpreted to originate from the participant volume \( (R_T \approx R_{\text{proj}}) \).
4. In \(^{16}\text{O} + \text{C}\) central collisions the average transverse radius appears to be considerably larger than the radii of the involved nuclei.
5. Longitudinal radii show an increase with target size for the lighter systems, but the value for \(^{16}\text{O} + \text{Au}\) is much smaller.
6. For \(^{16}\text{O} + \text{Au}\) central collisions pion pairs with \( Q_L \leq 10 \text{ MeV}/c \) show a transverse radius of \( R_T = 4.95 \pm 0.16 \text{ fm} \). Also in two-component fits a second very large component is observed \( (R_{T2} \geq 6 \text{ fm} \) and \( R_{L2} \geq 10 \text{ fm} \). This analysis emphasizes the need to study the dynamics of the so-called "spectator nucleons", which in fact seem to participate strongly in a later stage of the collision. Care should be taken, if one compares pion interferometry in different rapidity regions, when the presence of target spectators may obscure the interpretation.

Acknowledgements. This work was partially supported by the West German BMFT and DFG, the VW-Stifung, the United States DOE, the Swedish NFR and the Alexander von Humboldt Foundation. The contributions of K.G.R. Doss to the earlier analysis with the Plastic Ball at the Berkeley Bevalac are gratefully acknowledged.

Appendix

A new approach to the one-dimensional correlation function was used in this analysis. A set of "basis functions" \( \rho_i(r) \) was chosen for the source distribution. We have used the two choices A:

\[
\rho_i(r) = \exp \left( - \frac{r^2}{R_i^2} \right)
\]

and B:

\[
\rho_i(r) = \begin{cases} 
0 & r < r_{i-1} \\
1 & r_{i-1} \leq r \leq r_i \\
0 & r > r_i 
\end{cases}
\]

The source distribution was assumed to be a linear combination of these functions, where either A or B have been chosen:

\[
\rho(r) = \sum_{i=1}^{N} x_i \cdot \rho_i(r),
\]

which yields for the correlation function:

\[
C_2(Q) = 1 + \left| \sum_{i=1}^{N} x_i \cdot \tilde{\rho}_i(Q) \right|^2.
\]

The appropriate fourier transforms are (A):

\[
\tilde{\rho}_i(Q) = \exp \left( - \frac{Q^2 R_i^2}{4} \right)
\]

and (B)

\[
\tilde{\rho}_i(Q) = \frac{\sin (z_i Q') - \sin (z_{i-1} Q')}{Q'}
\]

where \( z_i = r_i/r_0 \) and \( Q' = r_0 \cdot Q \). This approach has the advantage that it can account for very different shapes in the correlation function, the range of accessible radii is however set by the user via the parameters \( R_i \) and \( r_i \). We have chosen \( R_1 = r_1 = 1 \text{ fm}, R_2 = r_2 = 2 \text{ fm}, \ldots R_5 = r_5 = 5 \text{ fm} \). These correlation functions are fitted by varying the \( x_i \). From the extracted fit parameters one can obtain rms-radii \( R_{\text{rms}} \) for the corresponding sources.

The value of \( \xi = \sum_{i=1}^{N} x_i \) gives an estimate of the fraction
of particles produced by chaotic sources in the sensitive range of the chosen basis functions. The missing fraction might e.g. come from larger sources that would not be accounted for by this approach. The intercept parameter of the usual analytical forms of the correlation function is related to the $x_i$ via:

$$\lambda = \bar{\xi}^2 = \left( \sum_{i=1}^{N} x_i \right)^2$$

References